FLUID MECHANICS

TUTORIAL No.7

FLUID FORCES

When you have completed this tutorial you should be able to

- Solve forces due to pressure difference.
- Solve problems due to momentum changes.
- Solve problems involving pressure and momentum changes.
- Solve forces on pipe bends.
- Solve problems on stationary vanes.
- Construct blade vector diagrams for moving vanes.
- Calculate the momentum changes over a moving vane.

Let's start by examining forces due to pressure changes.

1. PRESSURE FORCES

Consider a duct as shown in fig.1. First identify the control volume on which to conduct a force balance. The inner passage is filled with fluid with pressure p_1 at inlet and p_2 at outlet. There will be forces on the outer surface of the volume due to atmospheric pressure. If the pressures of the fluid are measured relative to atmosphere (i.e. use gauge pressures) then these forces need not be calculated and the resultant force on the volume is due to that of the fluid only. The approach to be used here is to find the forces in both the x and y directions and then combine them to find the resultant force.

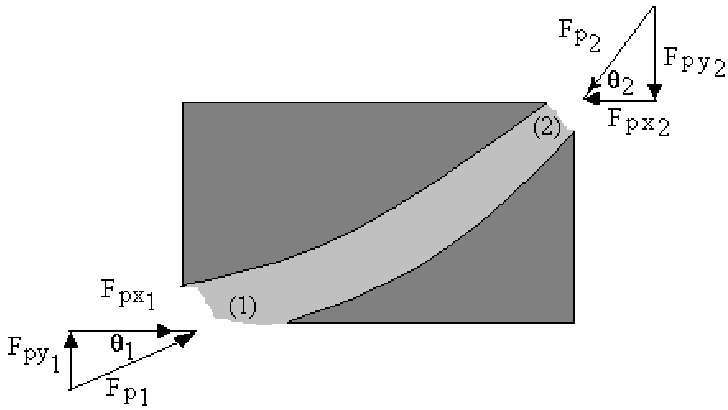


Fig. 1

The force normal to the plane of the bore is pA.

At the inlet (1) the force is $F_{p_1} = p_1A_1$

At the outlet (2) the force is $F_{p_2} = p_2A_2$

These forces must be resolved vertically and horizontally to give the following.

$F_{px1} = F_{p1} \cos \theta_1$ (to the right)

$F_{px2} = F_{p1} \cos \theta_2$ (to the left)

The total horizontal force $F_H = F_{px_1} - F_{px_2}$

$F_{py_1} = F_{p_1} \sin \theta_1$ (up)

$F_{py_2} = F_{p_2} \sin \theta_2$ (down)

The total vertical force $F_V = F_{py_1} - F_{py_2}$

WORKED EXAMPLE No. 1

A nozzle has an inlet area of 0.005 m^2 and it discharges into the atmosphere. The inlet gauge pressure is 3 bar. Calculate the resultant force on the nozzle.

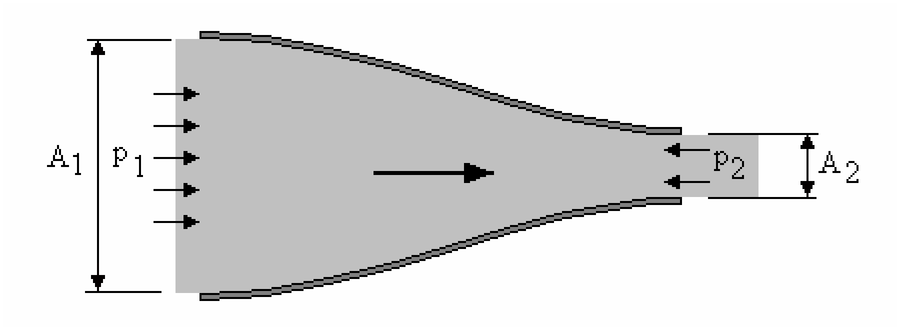


Fig.2

SOLUTION

Since the areas are only in the vertical plane, there is no vertical force. $F_V = 0$

Using gauge pressures, the pressure force at exit is zero. $F_{px2} = 0$

$F_{px1} = 3 \times 10^5 \times 0.005 = 1500$ N

$F_H = 1500 – 0 = 1500$ N to the right.

WORKED EXAMPLE No.2

The nozzle shown has an inlet area of 0.002 m^2 and an outlet area of 0.0005 m^2. The inlet gauge pressure is 300 kPa and the outlet gauge pressure is 200 kPa. Calculate the horizontal and vertical forces on the nozzle.

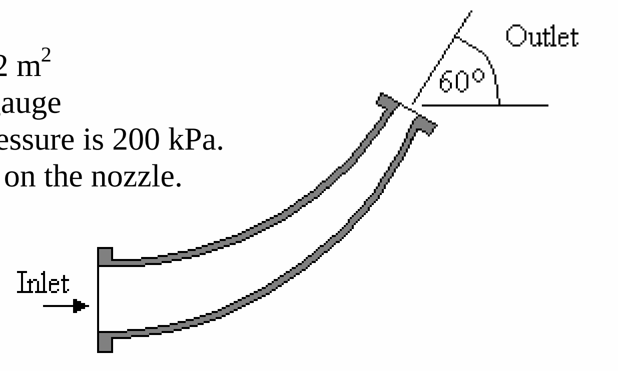


Fig.3

SOLUTION

$F_{p_1} = 300 \times 10^3 \times 0.002 = 600$ N

$F_{px_1} = 600$ N

$F_{py_1} = 0$ N since the plane is vertical.

$F_{p_2} = 200 \times 10^3 \times 0.0005 = 100$ N

$F_{px_2} = 100 \times \cos 60^o = 50$ N

$F_{py_2} = 100 \times \sin 60^o = 86.67$ N

Total Horizontal force $F_H = 600 - 50 = 550$ N

Total vertical force $F_V = 0 - 86.67 \text{ N} = - 86.67$ N

2. MOMENTUM FORCES

When a fluid speeds up or slows down, inertial forces come into play. Such forces may be produced by either a change in the magnitude or the direction of the velocity since either change in this vector quantity produces acceleration.

For this section, we will ignore pressure forces and just study the forces due to velocity changes.

2.1 NEWTON'S 2nd LAW OF MOTION

This states that ***the change in momentum of a mass is equal to the impulse given to it.***

Impulse = Force x time
Momentum = mass x velocity
Change in momentum = Δmv

Newton's second law may be written as $\Delta mv = Ft$

Rearrange to make F the subject. $\Delta mv/t = F$

Since $\Delta v/t$ = acceleration 'a' we get the usual form of the law $F = ma$

The mass flow rate is m/t and at any given moment this is dm/dt or m' and for a constant flow rate, only the velocity changes.

In fluids we usually express the second law in the following form. $F = (m/t)\,\Delta v = m'\Delta v$
$m'\Delta v$ is the rate of change of momentum so the second law may be restated as

F = Rate of change of momentum

F is the impulsive force resulting from the change. Δv is a vector quantity.

2.2 APPLICATION TO PIPE BENDS

Consider a pipe bend as before and use the idea of a control volume.

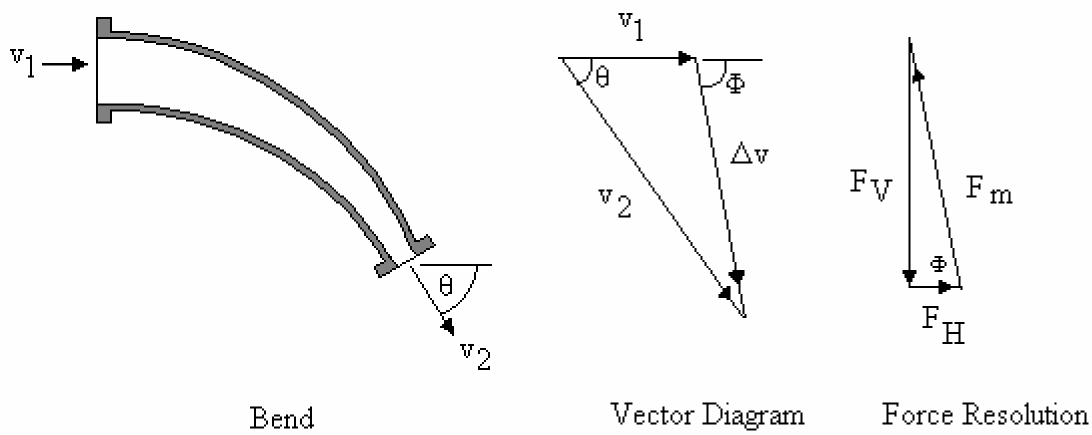


Fig.4

First find the vector change in velocity using trigonometry.

$$\tan\Phi = \frac{v_2\sin\theta}{v_2\cos\theta - v_1} \qquad \Delta v = \left\{(v_2\sin\theta)^2 + (v_2\cos\theta - v_1)^2\right\}^{\frac{1}{2}}$$

Alternatively Δv could be found by drawing the diagram to scale and measuring it.

If we had no change in magnitude then $v_1 = v_2 = v$ then **$\Delta v = v\,\{2(1 - \cos\theta)\}^{½}$**

The momentum force acting on the fluid is $F_m = m'\Delta v$

The force is a vector quantity which must be in the direction of Δv. Every force has an equal and opposite reaction so there must be a force on the bend equal and opposite to the force on the fluid. This force could be resolved vertically and horizontally such that

$F_H = F_m\cos\Phi$ and $F_V = F_m\sin\Phi$

This theory may be applied to turbines and pump blade theory as well as to pipe bends.

SELF ASSESSMENT EXERCISE No.1

1. A pipe bends through an angle of 90^o in the vertical plane. At the inlet it has a cross sectional area of 0.003 m^2 and a gauge pressure of 500 kPa. At exit it has an area of 0.001 m^2 and a gauge pressure of 200 kPa.

   Calculate the vertical and horizontal forces due to the pressure only.
   (Answers 200 N and 1500 N).

2. A pipe bends through an angle of 45^o in the vertical plane. At the inlet it has a cross sectional area of 0.002 m^2 and a gauge pressure of 800 kPa. At exit it has an area of 0.0008 m^2 and a gauge pressure of 300 kPa.

   Calculate the vertical and horizontal forces due to the pressure only.
   (Answers 169.7 N and 1430 N).

3. Calculate the momentum force acting on a bend of 130^o that carries 2 kg/s of water at 16m/s velocity.

   Determine the vertical and horizontal components. (Answers 24.5 N and 52.6 N)

4. Calculate the momentum force on a 180^o bend that carries 5 kg/s of water. The pipe is 50 mm bore diameter throughout. The density is 1000 kg/m^3.
   (Answer 25.46 N)

5. A horizontal pipe bend reduces from 300 mm bore diameter at inlet to 150 mm diameter at outlet. The bend is swept through 50^o from its initial direction.
   The flow rate is 0.05 m^3/s and the density is 1000 kg/m^3. Calculate the momentum force on the bend and resolve it into two perpendicular directions relative to the initial direction.
   (Answers 108.1 N and 55.46 N).

3. COMBINED PRESSURE AND MOMENTUM FORCES

Now we will look at problems involving forces due to pressure changes and momentum changes at the same time. This is best done with a worked example since we have covered the theory already.

WORKED EXAMPLE No.3

A pipe bend has a cross sectional area of 0.01 m^2 at inlet and 0.0025 m^2 at outlet. It bends 90^o from its initial direction. The velocity is 4 m/s at inlet with a pressure of 100 kPa gauge. The density is 1000 kg/m^3. Calculate the forces acting parallel and perpendicular to the initial direction.

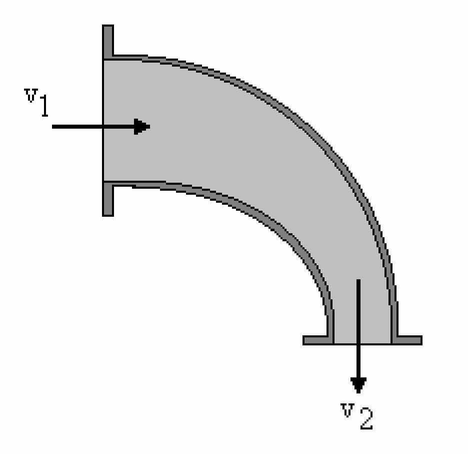


Fig. 5

SOLUTION

v_1 = 4m/s. Since $\rho A_1 v_1 = \rho A_2 v_2$ then v_2 = 16 m/s

We need the pressure at exit. This is done by applying Bernoulli between (1) and (2) as follows.

$$p_1 + ½ \rho v_1^2 = p_2 + ½ \rho v_2^2$$

$$100 \times 10^3 + ½\ 1000 \times 4^2 = p_2 + 1000 \times ½\ 16^2$$

$$p_2 = 0 \text{ kPa gauge}$$

Now find the pressure forces.

$F_{px_1} = p_1 A_1 = 1200$ N

$F_{py_2} = p_2 A_2 = 0$ N Next solve the momentum forces.

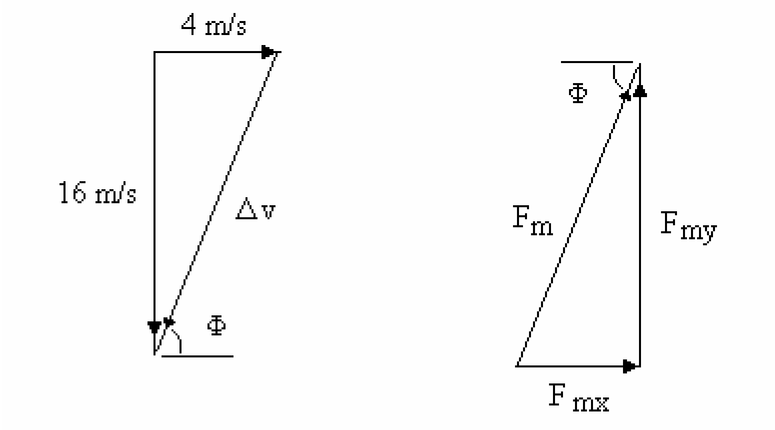


Fig.6

$m' = \rho A v = 40$ kg/s
$\Delta v = (4^2 + 16^2)^{1/2} = 16.49$ m/s
$F_m = m'\Delta v = 659.7$ N
$\phi = \tan^{-1}(16/4) = 75.96^o$

RESOLVE

$F_{my} = 659.7 \sin 75.96 = 640$ N
$F_{mx} = 659.7 \cos 75.96 = 160$ N

Total forces in x direction = 1200 + 160 = 1360 N
Total forces in y direction = 0 + 640 = 640 N

ALTERNATIVE SOLUTION

Many people prefer to solve the complete problems by solving pressure and momentum forces in the x or y directions as follows.

x direction $m'v_1 + p_1A_1 = F_X = 1200$ N

y direction $m'v_2 + p_2A_2 = F_Y = 640$ N

When the bend is other than 90^o this has to be used more carefully because there is an x component at exit also.

4. APPLICATIONS TO STATIONARY VANES

When a jet of fluid strikes a stationary vane, the vane decelerates the fluid in a given direction. Even if the speed of the fluid is unchanged, a change in direction produces changes in the velocity vectors and hence momentum forces are produced. The resulting force on the vane being struck by the fluid is an ***impulsive force.*** Since the fluid is at atmospheric pressure at all times after leaving the nozzle, there are no forces due to pressure change.

4.1 FLAT PLATE NORMAL TO JET

Consider first a jet of liquid from a nozzle striking a flat plate as shown in figure 7.

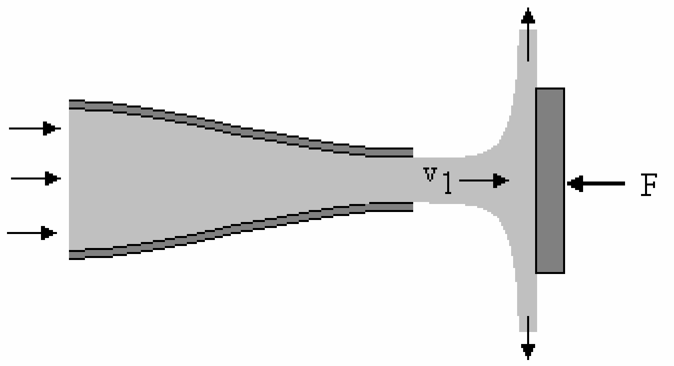


Fig.7

The velocity of the jet leaving the nozzle is v_1. The jet is decelerated to zero velocity in the original direction. Usually the liquid flows off sideways with equal velocity in all radial directions with no splashing occurring. The fluid is accelerated from zero in the radial directions but since the flow is equally divided no resultant force is produced in the radial directions. This means the only force on the plate is the one produced normal to the plate. This is found as follows.

m' = mass flow rate.
Initial velocity = v_1.
Final velocity in the original direction = $v_2 = 0$.
Change in velocity $=\Delta v = v_2 - v_1 = - v_1$
Force = $m'\Delta v = -mv_1$

This is the force required to produce the momentum changes in the fluid. The force on the plate must be equal and opposite so

$$\mathbf{F = m'v_1 = \rho A\, v_1}$$

WORKED EXAMPLE No.4

A nozzle has an exit diameter of 15 mm and discharges water into the atmosphere. The gauge pressure behind the nozzle is 400 kPa. The coefficient of velocity is 0.98 and there is no contraction of the jet. The jet hits a stationary flat plate normal to its direction. Determine the force on the plate. The density of the water is 1000 kg/m^3. Assume the velocity of approach into the nozzle is negligible.

SOLUTION

The velocity of the jet is $v_1 = C_V(2\Delta p/\rho)^{½}$

$$v_1 = 0.98\ (2x\ 400\ 000/1000)^{½} = 27.72\ \text{m/s}$$

The nozzle exit area $A = \pi \times 0.015^2/4 = 176.7 \times 10^{-6}\ \text{m}^2$.

The mass flow rate is $\rho A v_1 = 1000 \times 176.7 \times 10^{-6} \times 27.72 = 4.898$ kg/s.

The force on the vane = 4.898 x 27.72 = **135.8 N**

4.2 FLAT PLATE AT ANGLE TO JET

If the plate is at an angle as shown in fig. 8 then the fluid is not completely decelerated in the original direction but the radial flow is still equal in all radial directions. All the momentum normal to the plate is destroyed. It is easier to consider the momentum changes normal to the plate rather than normal to the jet.

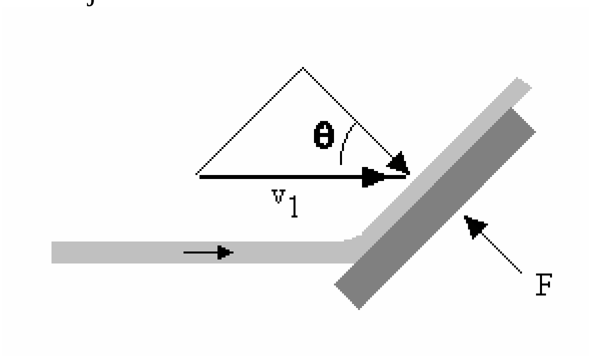


Fig.8

Initial velocity normal to plate = $v_1 \cos\theta$.
Final velocity normal to plate = 0.
Force normal to plate = $m'\Delta v = 0 - \rho A\ v_1 \cos\theta$.
This is the force acting on the fluid so the force on the plate is

$$\mathbf{m'\ v_1 \cos\theta \quad or \quad \rho A\ v_1^2 \cos\theta.}$$

If the horizontal and vertical components of this force are required then the force must be resolved.

WORKED EXAMPLE No. 5

A jet of water has a velocity of 20 m/s and flows at 2 kg/s. The jet strikes a stationary flat plate. The normal direction to the plate is inclined at 30^o to the jet. Determine the force on the plate in the direction of the jet.

SOLUTION

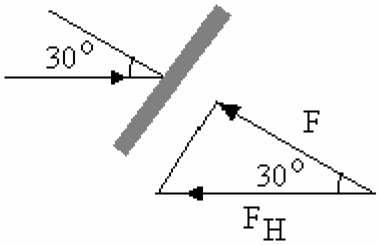


Fig.9

The force normal to the plate is $mv_1 \cos\theta = 2 \times 20\cos 30^o = 34.64$ N.

The force in the direction of the jet is found by resolving.

$F_H = F/\cos30^o = 34.64/\cos 30^o = 40$ N

4.3 CURVED VANES

When a jet hits a curved vane, it is usual to arrange for it to arrive on the vane at the same angle as the vane. The jet is then diverted from with no splashing by the curve of the vane. If there is no friction present, then only the direction of the jet is changed, not its speed.

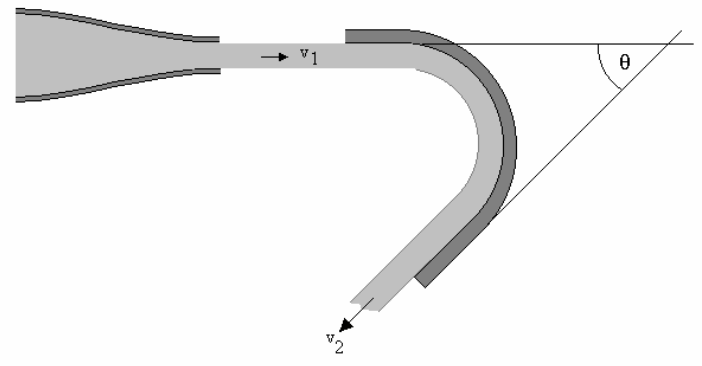
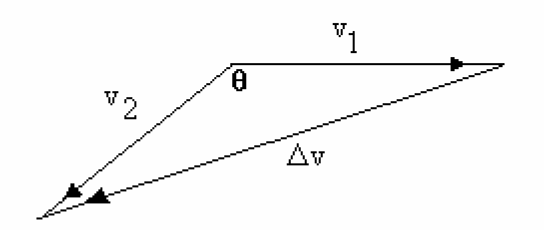


Fig.10

This is the same problem as a pipe bend with uniform size. v_1 is numerically equal to v_2.

Fig.11



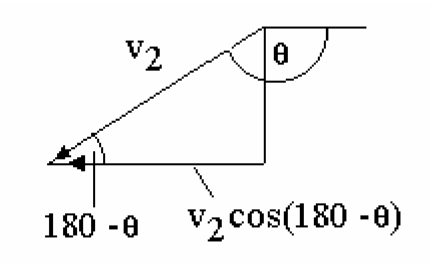
If the deflection angle is θ as shown in figs.10 and 11 then the impulsive force is

$$\mathbf{F = m'\Delta v = m'\, v_1\{2(1 - \cos\theta)\}^{1/2}}$$

The direction of the force on the fluid is in the direction of Δv and the direction of the force on the vane is opposite. The force may be resolved to find the forces acting horizontally and/or vertically.

It is often necessary to solve the horizontal force and this is done as follows.

Fig.12



Initial horizontal velocity = $v_{H1} = v_1$

Final horizontal velocity = $v_{H2} = -v_2 \cos(180 - \theta) = v_2 \cos\theta$

Change in horizontal velocity = Δv_{H1}

Since $v_2 = v_1$ this becomes $\Delta v_h = \{v_2 \cos\theta - v_1\} = v_1\{\cos\theta - 1\}$

Horizontal force on fluid = $m'v_1\{\cos\theta - 1\}$

The horizontal force on the vane is opposite so

$$\mathbf{Horizontal\ force = m'\Delta v_H = m'v_1\{1 - \cos\theta\}}$$

WORKED EXAMPLE No.6

A jet of water travels horizontally at 16 m/s with a flow rate of 2 kg/s. It is deflected 130º by a curved vane. Calculate resulting force on the vane in the horizontal direction.

SOLUTION

The resulting force on the vane is $F = m' v_1\{2(1 - \cos\theta)\}^{½}$

$$F = 2 \times 16 \{2(1 - \cos 130^o)\}^{½} = 58 \text{ N}$$

The horizontal force is

$$F_H = m' v_1\{\cos\theta - 1\}$$
$$F_H = 2 \times 16 \times (1 - \cos 130)$$
$$F_H = 52.6 \text{ N}$$

SELF ASSESSMENT EXERCISE No. 2

Assume the density of water is 1000 kg/m^3 throughout.

1. A pipe bends through 90º from its initial direction as shown in fig.13. The pipe reduces in diameter such that the velocity at point (2) is 1.5 times the velocity at point (1). The pipe is 200 mm diameter at point (1) and the static pressure is 100 kPa. The volume flow rate is 0.2 m^3/s. Assume there is no friction. Calculate the following.

   a) The static pressure at (2).

   b) The velocity at (2).

   c) The horizontal and vertical forces on the bend F_H and F_V.

   d) The total resultant force on the bend.

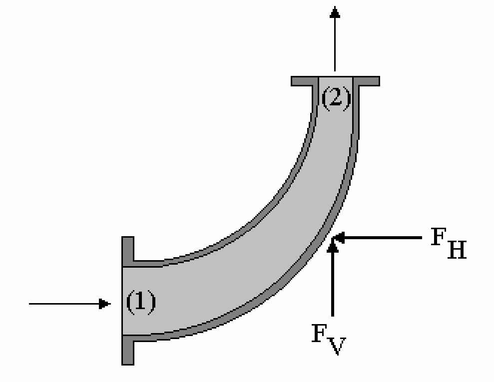


Fig.13

2. A nozzle produces a jet of water. The gauge pressure behind the nozzle is 2 MPa. The exit diameter is 100 mm. The coefficient of velocity is 0.97 and there is no contraction of the jet. The approach velocity is negligible. The jet of water is deflected 165^o from its initial direction by a stationary vane. Calculate the resultant force on the nozzle and on the vane due to momentum changes only.
(Answers 29.5 kN and 58.5 kN).

3. A stationary vane deflects 5 kg/s of water 50^o from its initial direction. The jet velocity is 13 m/s. Draw the vector diagram to scale showing the velocity change. Deduce by either scaling or calculation the change in velocity and go on to calculate the force on the vane in the original direction of the jet.
(Answer 49.8 N).

4. A jet of water travelling with a velocity of 25 m/s and flow rate 0.4 kg/s is deflected 150^o from its initial direction by a stationary vane. Calculate the force on the vane acting parallel to and perpendicular to the initial direction.
(Answers 18.66 N and 5 N)

5. A jet of water discharges from a nozzle 30 mm diameter with a flow rate of 15 dm^3/s into the atmosphere. The inlet to the nozzle is 100 mm diameter. There is no friction nor contraction of the jet. Calculate the following.

   i. The jet velocity.(21.22 m/s)
   ii. The gauge pressure at inlet. (223.2 kPa)
   iii. The force on the nozzle. (2039 N)

   The jet strikes a flat stationary plate normal to it. Determine the force on the plate.
   (312 N)

5. MOVING VANES

When a vane moves away from the jet as shown on fig.14, the mass flow arriving on the vane is reduced because some of the mass leaving the nozzle is producing a growing column of fluid between the jet and the nozzle. This is what happens in turbines where the vanes are part of a revolving wheel. We need only consider the simplest case of movement in a straight line in the direction of the jet.

5.1 MOVING FLAT PLATE

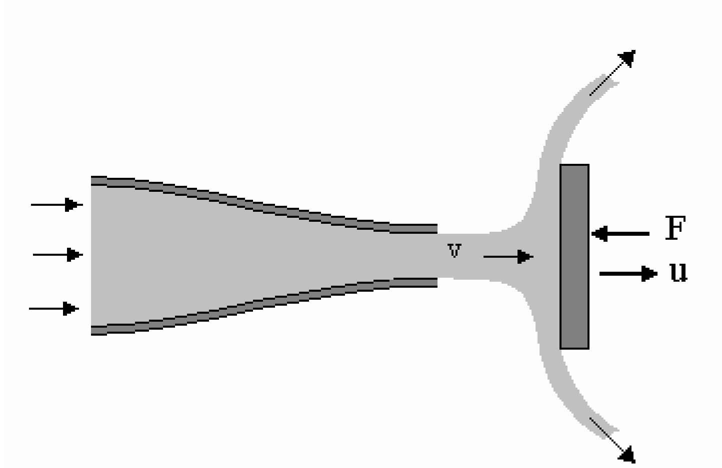


Fig.14

The velocity of the jet is v and the velocity of the vane is u. If you were on the plate, the velocity of the fluid arriving would be v - u. This is the relative velocity, that is, relative to the plate. The mass flow rate arriving on the plate is then

$$\mathbf{m' = \rho A(v-u)}$$

The initial direction of the fluid is the direction of the jet. However, due to movement of the plate, the velocity of the fluid as it leaves the edge is not at 90^o to the initial direction. In order to understand this we must consider the fluid as it flows off the plate. Just before it leaves the plate it is still travelling forward with the plate at velocity u. When it leaves the plate it will have a true velocity that is a combination of its radial velocity and u. The result is that it appears to come off the plate at a forward angle as shown.

We are only likely to be interested in the force in the direction of movement so we only require the change in velocity of the fluid in this direction.

The initial forward velocity of the fluid = v
The final forward velocity of the fluid = u
The change in forward velocity = v-u
The force on the plate = m'ρv = m' (v-u)
Since m' = ρA(v-u) then the force on the plate is

$$\mathbf{F = \rho A(v-u)^2}$$

5.2 MOVING CURVED VANE

Turbine vanes are normally curved and the fluid joins it at the same angle as the vane as shown in fig.15.

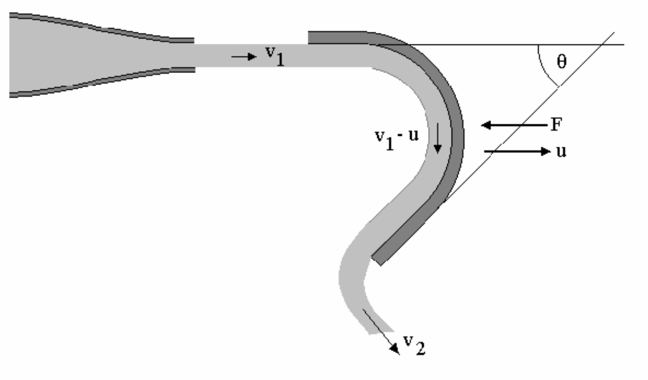


Fig.15

The velocity of the fluid leaving the nozzle is v_1. This is a true or absolute velocity as observed by anyone standing still on the ground. The fluid arrives on the vane with relative velocity v_1-u as before. This is a relative velocity as observed by someone moving with the vane. If there is no friction then the velocity of the fluid over the surface of the vane will be v_1-u at all points. At the tip where the fluid leaves the vane, it will have two velocities. The fluid will be flowing at v_1-u over the vane but also at velocity u in the forward direction. The true velocity v_2 at exit must be the vector sum of these two.

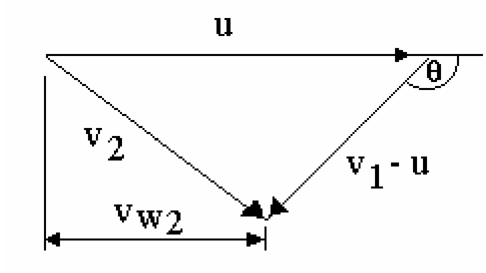


Fig.16

If we only require the force acting on the vane in the direction of movement then we must find the horizontal component of v_2. Because this direction is the direction in which the vane is whirling about the centre of the wheel, it is called the velocity of whirl v_{w2}. The velocity v_1 is also in the direction of whirling so it follows that $v_1 = v_{w1}$.

V_{w2} may be found by drawing the vector diagram (fig.16) to scale or by using trigonometry. In this case you may care to show for yourself that $v_{w2} = u + (v_1-u)(\cos\theta)$

The horizontal force on the vane becomes $F_H = m' (v_{w1}-v_{w2}) = m' (v_1-v_{w2})$

You may care to show for yourself that this simplifies down to $Fh = m'(v_1-u)(1-\cos\theta)$

This force moves at the same velocity as the vane. The power developed by a force is the product of force and velocity. This is called the Diagram Power (D.P.) and the diagram power developed by a simple turbine blade is D.P. $= m'u(v_1-u)(1-\cos\theta)$

This work involving the force on a moving vane is the basis of turbine problems and the geometry of the case considered is that of a simple water turbine known as a Pelton Wheel. You are not required to do this in the exam. It is unlikely that the examination will require you to calculate the force on the moving plate but the question in self assessment exercise 5 does require you to calculate the exit velocity v_2.

WORKED EXAMPLE No.7

A simple turbine vane as shown in fig.15 moves at 40 m/s and has a deflection angle of 150^o. The jet velocity from the nozzle is 70 m/s and flows at 1.7 kg/s.

Calculate the absolute velocity of the water leaving the vane and the diagram power.

SOLUTION

Drawing the vector diagram (fig.16) to scale, you may show that v_2 = 20.5 m/s. This may also be deduced by trigonometry. The angle at which the water leaves the vane may be measured from the diagram or deduced by trigonometry and is 46.9^o to the original jet direction.

D.P. = $m'u(v_1-u)(1+\cos\theta) = 1.7 \times 40(70-40)(1 - \cos 150) = 3807$ Watts

SELF ASSESSMENT EXERCISE No.3

1. A vane moving at 30 m/s has a deflection angle of 90^o. The water jet moves at 50 m/s with a flow of 2.5 kg/s. Calculate the diagram power assuming that all the mass strikes the vane.
   (Answer 1.5 kW).

2. Figure 10 shows a jet of water 40 mm diameter flowing at 45 m/s onto a curved fixed vane. The deflection angle is 150^o. There is no friction. Determine the magnitude and direction of the resultant force on the vane. (4916 N)

   The vane is allowed to move away from the nozzle in the same direction as the jet at a velocity of 18 m/s. Draw the vector diagram for the velocity at exit from the vane and determine the magnitude and direction of the velocity at exit from the vane. (14.53 m/s)